A SEMI-LAGRANGIAN AMR SCHEME FOR 2D TRANSPORT PROBLEMS IN CONSERVATION FORM

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Abstract. We propose a numerical instrument to solve, in dimension $N \in \{1, 2\}$, transport problems written in conservation form

$$\frac{\partial u}{\partial t} + \operatorname{div}_{\boldsymbol{x}} [\boldsymbol{a}(t, \boldsymbol{x})u] = 0, \qquad u(0, \boldsymbol{x}) = u^0(\boldsymbol{x}), \qquad (t, \boldsymbol{x}) \in \mathbb{R}_{\geq 0} \times \Omega, \tag{1}$$

where $\Omega = \prod_{n=1}^{N} [(x_n)_{\min}, (x_n)_{\max}] \subseteq \mathbb{R}^N$, $u : \mathbb{R}_{\geq 0} \times \Omega \to \mathbb{R}$ and $a : \mathbb{R}_{\geq 0} \times \Omega \to \mathbb{R}^N$. The amount $u(t, \boldsymbol{x})$ evolves following the laws described by the advection field $\boldsymbol{a}(t, \boldsymbol{x})$, whose expression depends on the nature of the system studied. Common issues in the simulation of such problems are the appearance or movement of large gradients, the filamentation of the phase space or the presence of vortices, in which cases many discretization points are required, while smooth zones can be given less resolution. If a Fixed Mesh (FM) discretization is used, then the choice of meshing the whole domain at the highest resolution is forced, which makes the numerical method time-consuming. Adaptive-Mesh-Refinement (AMR) [3, 1] schemes describe different zones of the domain Ω with different resolutions; the grid hierarchy is updated after each time step depending on the features of $u(t, \boldsymbol{x})$. The transport stages of (1) are solved by means of a semi-Lagrangian (SL) strategy based on integrating at the feet of the characteristics through the Point-Value Weighted Essentially Non Oscillatory (PV-WENO) scheme in order to avoid adding spurious, non-physical oscillations [2]. We extend to the 2D setting by making the time integration *dimension-by-dimension* thanks to a Strang splitting [4]. We discuss the quality of the results and the speedup with respect to a Fixed Mesh (FM) strategy through the following benchmark tests: in 1D, constant and variable-coefficient advections; in 2D, the Vlasov-Poisson system, the swirling deformation flow and the guiding-center model.

Keywords: Adaptive Mesh Refinement, WENO, semi-Lagrangian, Strang splitting, Vlasov-Poisson, guiding-center

Mathematics Subject Classifications (2000): 35L65, 35L67, 65M50, 65M55, 65M25

References

[1] Antonio Baeza, Anna Martínez-Gavara, and Pep Mulet. Adaptation based on interpolation errors for high order mesh refinement methods applied to conservation laws. *Applied Numerical Mathematics*, 62(4):278 – 296, 2012. Third Chilean Workshop on Numerical Analysis of Partial Differential Equations (WONAPDE 2010).

[2] J. A. Carrillo and F. Vecil. Nonoscillatory interpolation methods applied to Vlasov-based models. *SIAM J. Sci. Comput.*, 29(3):1179–1206 (electronic), 2007.

[3] A. Cohen, S. M. Kaber, S. Müller, and M. Postel. Fully adaptive multiresolution finite volume schemes for conservation laws. *Mathematics of Computations*, 72(241):183–225, 2003.

[4] Gilbert Strang. On the construction and comparison of difference schemes. *SIAM Journal on Numerical Analysis*, 5(3):pp. 506–517, 1968.

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